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May, 1989

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## 1. INTRODUCTION

This paper presents a model of endogenous stochastic growth, based on Schumpeter's idea of creative destruction. We begin from the belief, supported by many empirical studies starting with Solow (1957), that a large proportion of economic growth in developed countries is attributable to improvement in technology rather than the accumulation of capital. The paper models technological progress as occurring in the form of innovations, which in turn result from the activities of research firms.

We depart from existing models of endogenous growth (Romer, 1986, 1988; and Lucas, 1988) in two fundamental respects. First, we emphasize the fact that technological progress creates losses as well as gains, by rendering obsolete old skills, goods, markets, and manufacturing processes. This has both positive and normative implications for growth. In positive terms, the prospect of a high level of research in the future can deter research today by threatening the fruits of that research with rapid obsolescence. In normative terms, obsolescence creates a negative externality from innovations, and hence a tendency for *laissez-faire* to generate too much growth.

Obsolescence does not fit well into existing models of endogenous growth. Those models have only positive externalities, in the form of technology spillovers, and thus tend to generate too little growth. The closest in spirit to our model is that of Romer (1988). In that model innovations consist of the invention of new intermediate goods, neither better nor worse than existing ones. Once invented a good remains in production forever. Growth takes place because of the lengthening of the list of available intermediate goods. This model does a good job of capturing the division-of-labor aspect of growth. But adding obsolescence, by allowing old goods to be displaced by the introduction of new goods, may eliminate growth in this kind of model (see Deneckere and Judd, 1986<sup>1</sup>).

Our second departure is that we view the growth process as discontinuous. We share the view taken by many economic historians that individual technological breakthroughs have aggregate effects, that the uncertainty of innovations does not average out across industries.

This view is supported by recent empirical evidence of authors such as Nelson and Plosser (1982) and Campbell and Mankiw (1987) that the trend component of an economy's GNP includes a substantial random element.<sup>2</sup>

Existing models of endogenous growth do not produce a random trend, unless exogenous technology shocks are added to the model, as is done by King and Rebelo (1988). We take the view that technology shocks should not be regarded as exogenous in an analysis that seeks to explain the economic decisions underlying the accumulation of knowledge. Instead, we suppose that they are entirely the result of such decisions. Statistical innovations in GNP are produced by economic innovations, the distribution of which is determined by the equilibrium amount of research. This makes the distribution of technology shocks endogenous to the model.<sup>3</sup>

In making both of these departures from recent models of endogenous growth we take our inspiration from Schumpeter (1942, p.83, his emphasis):

The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets,...[This process] incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism.

In Schumpeter's view, capitalist growth is inherently uncertain. Fundamental breakthroughs are the essence of the process, and they affect the entire economy. However, the uncertainty is endogenous to the system, because the probability of a breakthrough depends upon the level of research, which in turn depends upon the the monopoly rents that constitute "the prizes offered by capitalist society to the successful innovator." (1942, p.102) Thus, "economic life itself changes its own data, by fits and starts." (1934, p.62).

The present paper develops a simple model that articulates these basic Schumpeterian elements. Although it is quite special in some dimensions we believe it is also simple and flexible enough to serve as a prototype model of growth through creative destruction.<sup>4</sup>

## 2. THE BASIC MODEL

### a. Assumptions

There are four classes of tradeable objects: land, labor, a consumption good, and a continuum of intermediate goods  $i \in [0,1]$ . There is also a continuum of identical infinitely-lived individuals, with mass  $N$ , each endowed with a one-unit flow of labor, and each with identical intertemporally additive preferences defined over lifetime consumption, with the constant rate of time preference  $r > 0$ . Except in section 5 below, we assume a constant marginal utility of consumption at each date; thus  $r$  is also the unique rate of interest in the economy. There is no disutility from supplying labor. There is also a fixed supply  $H$  of land.

The consumption good is produced using land and the intermediate goods, subject to constant returns. Since  $H$  is fixed we can express the production function as:

$$(2.1) \quad y = \int_0^1 \{F[x(i)]/c(i)\} di$$

where  $F' > 0$ ,  $F'' < 0$ ,  $y$  is the flow output of consumption good,  $x(i)$  the flow of intermediate input  $i$ , and  $c(i)$  a parameter indicating, for given factor prices, the unit cost of producing the consumption good using the intermediate input  $i$ .

Each intermediate good  $i$  is produced using labor alone, according to the linear technology:

$$(2.2) \quad x(i) = L(i)$$

where  $L(i)$  is the flow of labor used in intermediate sector  $i$ .

Labor has an alternative use to producing intermediate goods. It can also be used in research, which produces a random sequence of innovations. The Poisson arrival rate of innovations in the economy at any instant is  $\lambda n$ , where  $n$  is the flow of labor used in research and  $\lambda$  a constant parameter given by the technology of research. There is no memory in this

technology, since the arrival rate depends only upon the current flow of input to research, not upon past research.<sup>5</sup>

Time is continuous, and indexed by  $\tau \geq 0$ . The symbol  $t = 0, 1, \dots$  denotes the interval starting with the  $t^{\text{th}}$  innovation (or with  $\tau = 0$  in the case of  $t = 0$ ) and ending just before the  $t+1^{\text{st}}$ . The length of each interval is random. If the constant flow of labor  $n_t$  is applied to research in interval  $t$ , its length will be exponentially distributed with parameter  $\lambda n_t$ .

Each innovation consists of the invention of a new line of intermediate goods, whose use as inputs allows more efficient methods to be used in producing consumption goods. We have in mind such "input" innovations<sup>6</sup> as the steam engine, the airplane, and the computer, whose use made possible new methods of production in mining, transportation, and banking. An innovation need, not, however, be as revolutionary as these examples, but might consist instead of a new generation of intermediate goods, similar to the old ones.

Specifically, use of the new line of intermediate goods reduces the cost parameters  $c(i)$  in (2.1) by the factor  $\gamma \in (0, 1)$ . We assume away all lags in the diffusion of technology.<sup>7</sup> The most modern intermediate good is always produced in each sector, and the unit-cost parameter during interval  $t$  is the same for all sectors; thus:

$$(2.3) \quad c_t(i) = c_t = c_0 \gamma^t \quad \forall i \in [0, 1], t = 0, 1, \dots,$$

where  $c_0$  is the initial value given by history. (Of course, it is always possible to produce the consumption good using an old technology, with a correspondingly old line of intermediate goods.)<sup>8</sup>

A successful innovator obtains a continuum of patents, one for each intermediate sector, each one granting the holder the exclusive right to produce the newly invented intermediate good in that sector. We assume, however, that anti-trust laws prohibit anyone from retaining that right in a positive measure of sectors. So the innovator sells each patent to a firm that becomes the local monopolist in that sector until the next innovation occurs. (We assume perfect competition in all markets other than those for the intermediate goods and for patents.)

The innovator offers each patent for sale at a price equal to the expected present value of the monopoly rents accruing to the patent. The buyer pays that price as a lump sum in exchange for unconstrained use of the patent. Thus we implicitly rule out royalties and other contingent contracts between the innovator and the local monopolist. The force of this restriction will be discussed in (d) below where it is argued that without the restriction the two parties would want a contract with negative royalties. The restriction might therefore be rationalized either by noting that negative royalties are not in fact observed, or by referring to costs of monitoring the output of the intermediate good.

No patent covers the use of a new intermediate good in the consumption—good sector. The idea behind this assumption is that potential uses of the new good are too obvious, even before its invention, to be patented. The assumption might also be rationalized by the observation that important innovations like the ones mentioned above have had widespread uses too numerous and diffuse for anyone to monopolize them.

#### **b. The Intermediate Monopolist's Decision Problem**

The local monopolist's objective is to maximize the expected present value of the flow of profits over the current interval. When the interval ends so do the rents. The only uncertainty in the decision problem arises because the length of the interval is random. Because no one operates in a positive measure of sectors the monopolist takes as given the amount of research at each time, and hence also takes as given the length of the interval. Because of this and because the cost of the patent is sunk, the monopolist's strategy will be simply to maximize the flow of operating profits  $\pi$  at each instant.

In equilibrium all prices and quantities are constant throughout the interval. Also, by symmetry, the same quantity  $x_t$  will be chosen by each monopolist. This quantity will also be the total output of intermediate goods in period  $t$ , which by (2.2) equals the total employment of labor in manufacturing; i.e.:



$$x_t = \int_0^1 x_t(i) di = \int_0^1 L_t(i) \cdot di$$

We can then re-express (2.1) as:

$$y_t = \frac{F(x_t)}{c_t}.$$

The inverse demand curve facing a monopolist charging the price  $p_t$  is the marginal product schedule:

$$(2.4) \quad p_t = F'(x_t)/c_t.$$

Thus the decision problem is to choose  $x_t$  so as to maximize  $[F'(x_t)/c_t - w_t]x_t$ , taking  $c_t$  and  $w_t$  as given. The necessary first-order condition is

$$(2.5) \quad c_t w_t = F'(x_t) + x_t F''(x_t).$$

Assume that the monopolist's marginal-revenue schedule is downward-sloping:

Assumption A.1:  $2F''(x) + x F'''(x) < 0 \quad \forall x > 0$

This condition holds automatically when  $F''' < 0$ ; we show in Appendix 1 that it also holds when  $F$  comes from a CES production function. It follows from (2.5) and A.1 that there is a unique solution to the decision problem:

$$(2.6) \quad x_t = \tilde{x}(c_t w_t)$$

where  $\tilde{x}$  is strictly decreasing. Thus the demand for labor in the intermediate industry is a decreasing function of the cost (in terms of the consumption good) of producing one efficiency unit of an intermediate good.

Likewise we can express the monopolist's price and flow of profits as:

$$(2.7) \quad p_t = \tilde{p}(c_t w_t)/c_t \equiv F'(\tilde{x}(c_t w_t))/c_t,$$

and

$$(2.8) \quad \begin{aligned} \pi_t &= \tilde{\pi}(c_t w_t)/c_t \\ &\equiv [(\tilde{p}(c_t w_t) - c_t w_t) \tilde{x}(c_t w_t)]/c_t \\ &= -[\tilde{x}(c_t w_t)]^2 F''(\tilde{x}(c_t w_t)) \end{aligned}$$

where  $\tilde{p}$  is strictly increasing and  $\tilde{\pi}$  is strictly decreasing.

For future use, we also assume:

Assumption A.2:  $\tilde{x}(0) = \infty$ ,  $\tilde{x}(\infty) = 0$ .

We have not allowed the local monopolist's decision problem to be constrained by potential competition from the holder of the previous patent. This is because if the constraint were potentially binding; i.e. if the innovation were non-drastic in the usual sense (see, for example Tirole, 1988, ch.10) then the current patent would be of greatest value to the holder of the previous patent, who would face no such competition. Thus our model predicts that non-drastic innovations would be implemented by incumbent producers, whereas drastic innovations would generally be implemented by new firms. This is in accordance with several empirical studies to the effect that incumbent firms implement less fundamental innovations than do new entrants. (See, for example, Scherer, 1980.)

Whether or not the innovation is drastic is determined by whether or not a competitive producer of the consumption good would incur a loss buying from the previous monopolists, at a price equal to the unit cost  $w_t$ . Thus the condition for a drastic innovation is

$$C(p_t^H, w_t) > c_t^{-1},$$

where  $C(\cdot)$  is the unit-cost function uniquely associated with the production function  $F$ , and  $p_t^H$  is the equilibrium price of land. The latter is determined by the conditions for competitive equilibrium in the markets for land and the consumption good:

$$C(p_t^H, p_t) = c_t^{-1}$$

where  $p_t$  is the equilibrium market price of all other intermediate goods, given by (2.7).

The special case of a Cobb–Douglas production is defined by:

$$(2.9) \quad F(x_t) = x_t^\alpha$$

It is straightforward to verify that in this case:

$$(2.10) \quad x_t = (c_t w_t / \alpha^2)^{1/(\alpha-1)},$$

$$(2.11) \quad p_t = w_t/\alpha,$$

$$(2.12) \quad \pi_t = [(1 - \alpha)/\alpha]w_t x_t,$$

and innovations will be drastic if and only if:

$$(2.13) \quad \gamma < \alpha^\alpha.$$

### c. Research

There are no contemporaneous spillovers in the research sector; that is, a research firm employing  $z$  will experience innovations with a Poisson arrival rate  $\lambda z$ , and these arrivals will be independent of other firms' research employment  $\bar{n}_t = n_t - z$ .<sup>9</sup> Let  $W_t$  be the value of a research firm after the  $t^{\text{th}}$  innovation, and let  $V_t$  be the value of patents from the  $t^{\text{th}}$  innovation. If the firm makes an innovation it will receive  $V_{t+1}$ . The density of this event at the current instant is  $\lambda z$ . Thus the expected payoff per unit time from successful innovation is  $\lambda z V_{t+1}$ . The firm will realize a capital gain on  $W_{t+1} - W_t$  when any firm makes an innovation. Thus its expected rate of capital gain is  $\lambda n_t (W_{t+1} - W_t)$ . Its flow of labor cost is  $w_t z$ . It takes  $w_t$  and  $\bar{n}_t$  as given. Thus we have the Bellman equation:<sup>10</sup>

$$(2.14) \quad rW_t = \max_{\{z \geq 0\}} \lambda z V_{t+1} + \lambda(\bar{n}_t + z)(W_{t+1} - W_t) - w_t z$$

Assume free entry into research. Then  $W_t = 0$ , so (2.14) can be expressed as:

$$(2.15) \quad 0 = \max_{\{z \geq 0\}} \lambda z V_{t+1} - w_t z$$

The Kuhn-Tucker condition is:

$$(2.16) \quad w_t \geq \lambda V_{t+1}, z \geq 0, \text{ with at least one equality (value).}$$

The value  $V_t$  is the expected present value of the constant flow  $\pi_t$  over an interval whose length is exponentially distributed with parameter  $\lambda n_t$ :

$$(2.17) \quad V_t = \frac{\pi_t}{r + \lambda n_t}.$$

Note that there is an important intertemporal spillover in this model. An innovation reduces costs forever. It allows each subsequent innovation to reduce the unit-cost parameter  $c_t$  by the same fraction  $\gamma$ , and with the same probability  $\lambda n_t$ , but from a starting value  $c_{t-1}$  that will be lower by the fraction  $\gamma$  than it would otherwise have been. The producer of an innovation will capture (some of) the rents from that cost reduction, but only during the next interval. After that the rents will be captured by other innovators, building upon the basis of the present innovation, but without compensating the present innovator.<sup>11</sup> This intertemporal spillover will play a role in the welfare analysis of section 4 below.

The model thus embodies Schumpeter's idea of "creative destruction". Each innovation is an act of creation aimed at capturing monopoly rents. But it also destroys the monopoly rents that motivated the previous creation. Creative destruction accounts for the term  $\lambda n_t$  in the denominator of  $V_t$  in (2.17). More research this period will reduce the expected tenure of the current monopolists, and hence reduce the expected present value of their flow of rents.<sup>12</sup>

#### d. Wage Determination

Because  $n_t$  is aggregate research employment, (2.16) implies:

$$(2.18) \quad w_t \geq \lambda V_{t+1}, \quad n_t \geq 0, \quad \text{w.a.l.o.e.}$$

Combining (2.8), (2.17), (2.18) and the equilibrium condition:

$$(2.19) \quad x_t + n_t = N; \quad t = 0, 1, \dots$$

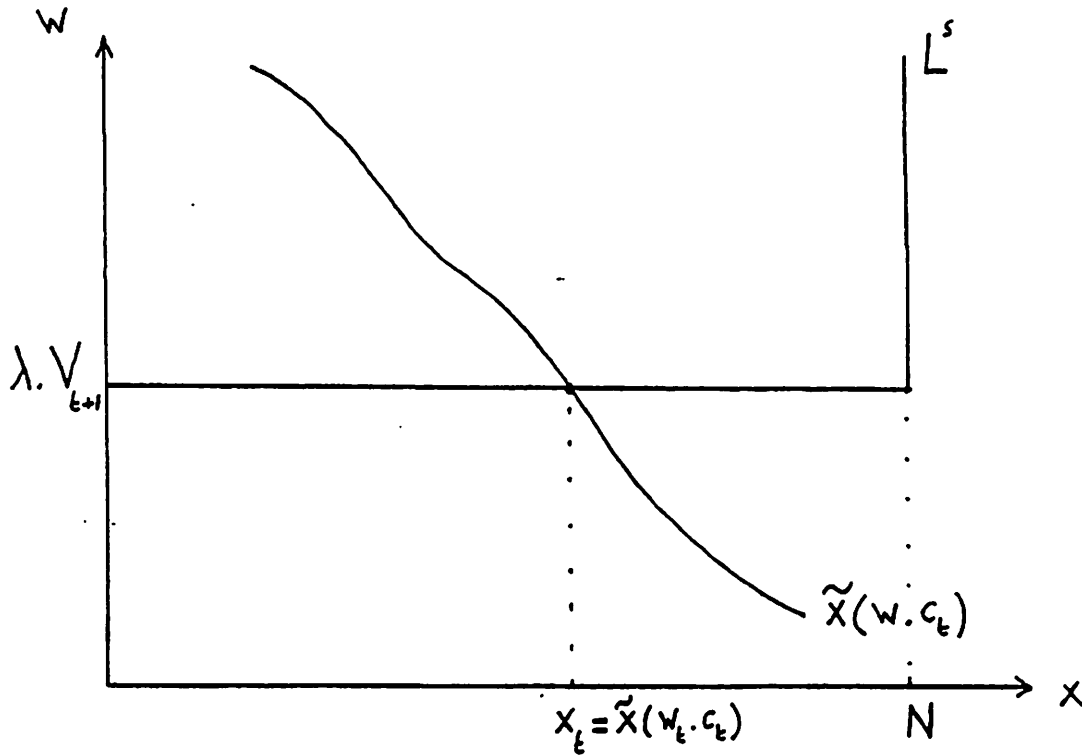
yields the condition:

$$(2.20) \quad w_t \geq \frac{\lambda \pi_{t+1}}{r + \lambda n_{t+1}} = \frac{\tilde{\pi}(c_{t+1} w_{t+1})/c_{t+1}}{(r/\lambda) + N - \tilde{x}(c_{t+1} w_{t+1})}, \quad x_t \leq N; \quad \text{w.a.l.o.e.}$$

Condition (2.20) describes the supply of labor to the intermediate sector. It says that unless that sector absorbs all the economy's labor, the wage will equal labor's opportunity cost in research,  $\lambda V_{t+1}$ . This condition, together with the labor-demand schedule (2.6), jointly

determine  $w_t$  and  $x_t$  in terms of  $w_{t+1}$  and  $x_{t+1}$ , as shown in Figure 1.

Figure 1



It is clear now why a successful innovator would want to offer a contract with negative royalties. Such a contract would induce each local monopolist to produce more than  $\tilde{x}(c_t, w_t)$ . This would have two effects on  $V_t$ , as given by (2.17). First, it would reduce the numerator below the maximal value given by (2.8). Second, it would also reduce the denominator by reducing  $n_t$ , through (2.19). In the neighborhood of the zero-royalty solution analyzed above the former effect would be a second-order small by the envelope theorem, but the latter would not. Thus the seller of the  $t^{\text{th}}$  line of patents would want negative royalties so as to discourage creative destruction during interval  $t$ . The local monopolists would not attempt to discourage creative destruction on their own by producing more than  $\tilde{x}(c_t, w_t)$  in the absence of negative royalties, because each one is too small to affect  $n_t$ .

### 3. PERFECT FORESIGHT EQUILIBRIA (PFE)

Using (2.6) and the fact that  $\tilde{x}$  is an invertible function, we can write  $w_t = \tilde{x}^{-1}(x_t)/c_t$ .

Multiplying both sides of (2.20) by  $c_t$  and using the identity:  $c_{t+1} = \gamma c_t$ , we obtain:

$$(3.1) \quad \tilde{x}^{-1}(x_t) \geq \frac{\tilde{\pi}(\tilde{x}^{-1}(x_{t+1}))/\gamma}{r/\lambda + N - x_{t+1}}; \quad x_t \leq N, \text{ waloge}$$

which we can re-express by the following first-order difference equation:

$$(3.2) \quad x_t = \mathcal{F}(x_{t+1}) = \min(G(x_{t+1}), N)$$

where:<sup>13</sup>

$$G(x_{t+1}) = \tilde{x} \left[ \frac{\tilde{\pi}(\tilde{x}^{-1}(x_{t+1}))/\gamma}{r/\lambda + N - x_{t+1}} \right].$$

We define a perfect foresight equilibrium (PFE) as a sequence  $\{x_t\}_0^\infty$  in  $[0, N]$  satisfying (3.2) for all  $t \geq 0$ . In PFE everyone can predict the future evolution of the endogenous variables  $(w_t, x_t, y_t, \pi_t, n_t, V_t)$  with certainty.<sup>14</sup> However, the length of each interval (in real time  $\tau$ ), as well as the identity of each innovator, is random. The characterization of PFE is simplified by the following:

Lemma 1: The mapping  $x \mapsto G(x)$  is decreasing in  $x$ .

Lemma 1 follows from the fact that both  $\tilde{x}$  and  $\tilde{\pi}$  are decreasing functions. The economic interpretation is as follows: A foreseen increase in  $x_{t+1}$  will raise the reward  $\lambda V_{t+1}$  to the next innovator: on the one hand it raises the flow of monopoly rents  $\tilde{\pi}(\tilde{x}^{-1}(x_{t+1}))$  and on the other hand it reduces the amount of research and hence the amount of creative destruction next interval (the effect in the denominator). The increase in  $\lambda V_{t+1}$  will raise the equilibrium wage  $w_t$  this period (if  $x_t < N$ ), which in turn will induce the intermediate monopolists to reduce their demand for labor this period,  $x_t$ .

Because (3.2) is forward-looking, history does not determine a unique value of  $x_0$  in PFE. Typically, there will be a continuum of PFE indexed by the initial value  $x_0$ . However there will only be a finite number of periodic trajectories to which any PFE could converge asymptotically. More precisely:

Proposition 1: All PFE are, or converge asymptotically to one of the following:

- a stationary equilibrium (SE) with positive growth,
- SE with zero growth,
- a "real" 2-cycle, or
- a "no-growth trap" (NGT).

Furthermore there always exists a unique SE.

We define SE as a steady state  $\hat{x} = \mathcal{J}(\hat{x})$ . In SE the economy experiences balanced growth in the sense that the allocation of employment between manufacturing ( $\hat{x}$ ) and research ( $\hat{n} = N - \hat{x}$ ) remains constant. Growth is positive if  $\hat{x} < N$  and zero if  $\hat{x} = N$ , because there will be innovations if and only if labor is allocated to research.

We define real 2-cycle as a pair  $(x^0, x^1)$  such that:

$$(3.3) \quad x^0 = \mathcal{J}(x^1), x^1 = \mathcal{J}(x^0), x^1 \neq x^0 \text{ and } N \notin \{x^0, x^1\}.$$

Then a real 2-cycle corresponds to PFE in which manufacturing employment oscillates between two different values with each succeeding innovation. High manufacturing employment in odd intervals raises the reward to research during even intervals, and hence depresses manufacturing employment during even intervals, through the mechanism discussed above. Likewise, low manufacturing employment in even intervals raises manufacturing employment in odd intervals.

A "no-growth" trap is a pair  $(x^0, x^1)$  such that the first three conditions of (3.3) hold, but  $N \in \{x^0, x^1\}$ . As shown in Figure 2 below, NGT exists when  $G(N) < G^{-1}(N) \equiv x_c$ .



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**a. Stationary Equilibrium (SE)**

In the Cobb–Douglas case, SE with positive growth is determined by:

$$\hat{x} = G(\hat{x}) = \left[ \frac{\frac{1-\alpha}{\alpha} \cdot \frac{\hat{x}^\alpha}{\gamma}}{r/\lambda + N - \hat{x}} \right]^{1/(\alpha-1)}$$

i.e. by the simple equation:

$$(3.4) \quad \gamma = \frac{\frac{1-\alpha}{\alpha} \cdot \hat{x}}{r/\lambda + N - \hat{x}}$$

For growth to be positive, it is necessary and sufficient to have:

$$(3.5) \quad \gamma < \frac{\lambda}{r} \cdot \frac{1-\alpha}{\alpha} \cdot N$$

In particular, for a fixed value of  $\gamma$ ,  $\lambda$ ,  $r$ ,  $N$ , a necessary and sufficient condition for positive growth is that the parameter  $\alpha$  be sufficiently small:  $\alpha < \alpha^* \equiv \lambda N / (\lambda N + \gamma r) < 1$ .

To interpret this result note that  $\alpha$  is an inverse measure of monopoly power in each intermediate sector. Specifically,  $1-\alpha$  is the Lerner (1934) measure of monopoly power (price minus marginal cost divided by price),  $(1-\alpha)^{-1}$  is the elasticity of demand faced by an intermediate monopolist, and  $1-\alpha$  is the fraction of equilibrium revenue in an intermediate sector accruing to the monopolist,  $\frac{\pi_t}{\pi_t + w_t x_t}$ . Thus, if monopoly power is too weak ( $\alpha \geq \alpha^*$ ) then the flow of monopoly rents from the next innovation would not be enough to induce positive research aimed at capturing those rents even if they could be retained forever, with no creative destruction in the next interval.

**b. "No-growth" Trap**

As we have seen, NGT will exist iff:

$$(3.6) \quad G(N) < x^c \equiv G^{-1}(N)$$

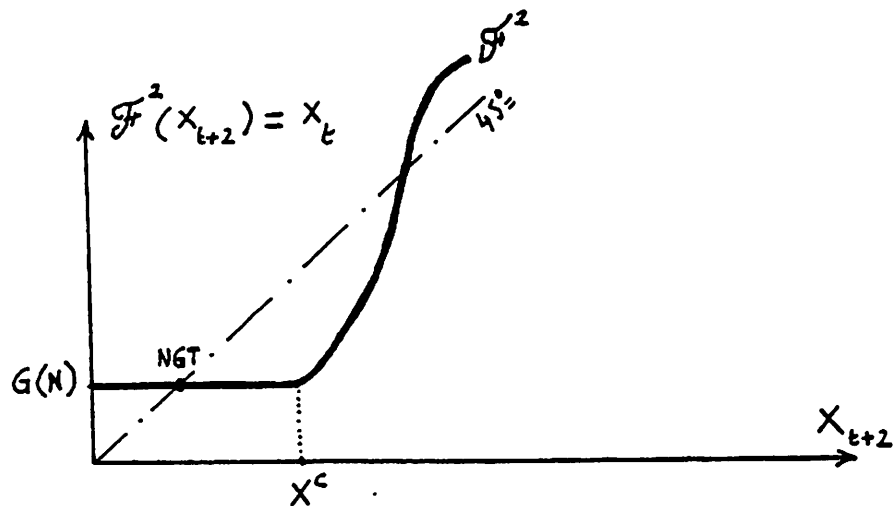


Figure 3

In particular NGT will always exist when the interest rate  $r$  is sufficiently small.<sup>16</sup> Note that the smaller  $r$  (or  $r/\lambda$ ), the easier it is to satisfy (3.5): in other words, "no-growth trap" equilibria are more likely to exist in an economy that possesses a positive balanced-growth equilibrium!

c. Real 2-cycle

A sufficient condition for a real 2-cycle to exist is that:  $G(N) < x^c$  (i.e. (3.6)) and

$\frac{dF^2}{dx}(\hat{x}) < 1$ , where  $\hat{x}$  is the unique SE.<sup>17</sup> (Figure 4 below).

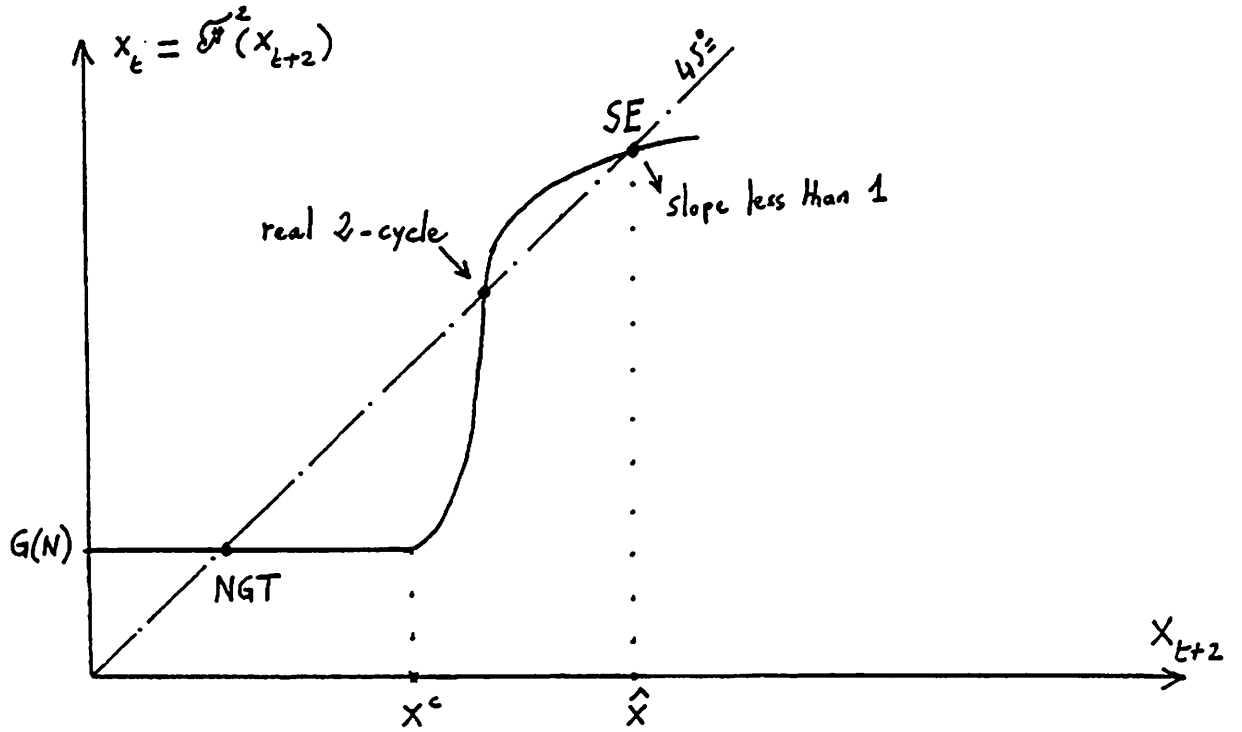


Figure 4

**Proposition 2:** When  $R = r/\lambda$  is sufficiently small and the elasticity of demand for intermediate input  $(\eta)^{18}$  is sufficiently close to 1, there exists a real 2-cycle.

The assumption that  $R$  is small guarantees that  $G(N) < x^c$ . (See (b) above). Also:

$$(3.7) \quad \mathcal{F}'(\hat{x}) = (\eta - 1) \cdot \left[ \frac{\gamma(1 - \eta)}{1 - \frac{1}{\eta} - \frac{x\eta}{\gamma}} - 1 \right]$$

(See Appendix 2). This expression approaches zero as  $\eta$  approaches 1. Proposition 2 then follows from the fact that:

$$\frac{d\mathcal{F}^2}{dx}(\hat{x}) = \mathcal{F}'(\hat{x}) \cdot \mathcal{F}'(\mathcal{F}(\hat{x})) = (\mathcal{F}'(\hat{x}))^2.$$

In the Cobb–Douglas case we have:

$$(3.8) \quad \eta = \frac{1}{1-\alpha}$$

Then, from Proposition 2, we know that there exists a real 2-cycle whenever  $\alpha$  is small and  $G(N) < x^c$  ( $r/\lambda$  small). We can summarize the conclusions obtained in the Cobb–Douglas case

by the following picture. Again  $\alpha$  can be interpreted as an inverse measure of the degree of monopoly power in the intermediate industry.

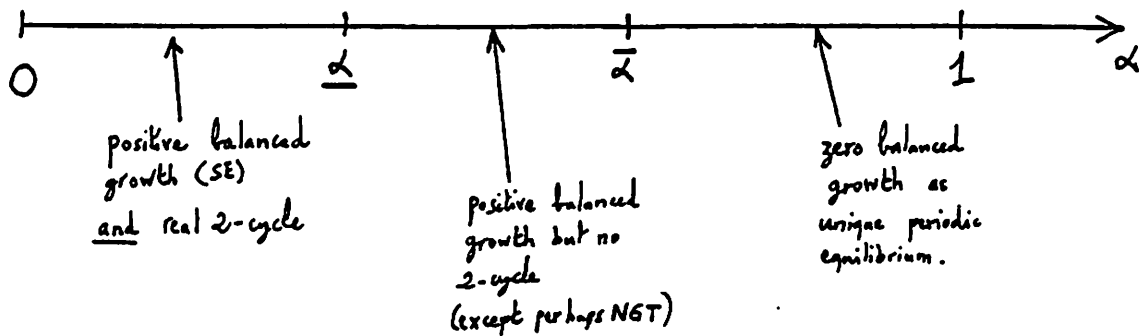


Figure 5

The reason for cycles in this model is similar, but not identical, to the reason for cycles in the model of Shleifer (1986).<sup>19</sup> In the Shleifer model innovations are exogenous, and there are multiple equilibrium strategies for implementing them, because of an aggregate-demand externality. That externality is present in this model in an intertemporal rather than intersectoral form. As (3.1) makes clear, higher manufacturing output next period raises the flow of monopoly rents next period. As we have seen, it is this effect, together with the effect of creative destruction, that makes  $x_t$  depend inversely upon  $x_{t+1}$ , and which therefore makes high manufacturing employment in odd intervals together with low manufacturing employment in even intervals a possible equilibrium configuration.

There are several questions that would need to be addressed before judging the likely empirical relevance of these 2-cycles. One is their observability under realistic expectational assumptions; equilibria that are stable under perfect foresight (as in Figure 4 above) are often unstable under learning (Grandmont and Laroque, 1986). Another is the size of the resulting output fluctuations; modern economies typically allocate no more than 2 or 3 percent of their

labor force to research. We regard these questions as open. Learning raises many unresolved issues, and even if the movement of labor between manufacturing and research would cause small proportional fluctuations in the level of output it could produce large proportional fluctuations in research, and hence in the growth rate of output. Rather than pursue these questions further we shall focus the rest of the paper on balanced growth equilibria. Accordingly, the main interest of the above analysis is not to propose a theory of cycles but to describe the logic of the model of creative destruction.

#### 4. BALANCED GROWTH

##### a. Time-Series Properties of Output

One of the benefits of endogenizing technical change is that we can endogenize the average rate of growth (AGR) of the economy. Rather than have AGR depend upon exogenous population growth and/or exogenous technical progress as in the neoclassical growth model, we have made it depend upon various factors that affect the incentive to do research and the fruitfulness of research in reducing production costs. In this sense we are following the seminal contributions of Romer (1986), Lucas (1988) and others. Another benefit is that we can endogenize the variability of the growth rate (VGR). The variance of what appear in the model as technology shocks depends upon the same economic factors that determine AGR. In the extreme case where the incentive to do research is so small as to result in no growth, VGR will also be zero.

We shall now proceed under the assumption that the economy is always in a situation of positive balanced growth. Thus:<sup>20</sup>

$$(4.1) \quad \hat{x} = \tilde{x} \left[ \frac{\lambda/\gamma \cdot \tilde{\pi}(\tilde{x}^{-1}(\hat{x}))}{r + \lambda(N - \hat{x})} \right] = G(\hat{x}) \text{ (more precisely } G(\hat{x}, r, \lambda, \gamma, N))$$

To see how a change in the parameters  $r, \lambda, \gamma, N$  affects the equilibrium level of research ( $\hat{n} = N - \hat{x}$ ), it suffices to determine the sign of the partial derivatives  $\mathcal{H}_r, \mathcal{H}_\lambda, \mathcal{H}_\gamma, \mathcal{H}_N, \mathcal{H}_{\hat{x}}$

where  $\mathcal{H}$  is defined by:

$$\mathcal{H}(\hat{x}, r, \lambda, \gamma, N) \equiv G(\hat{x}) - \hat{x}.$$

The following Lemma is an immediate consequence of Lemma 1 and the fact that  $\tilde{\pi}$  and  $\tilde{x}$  are decreasing:

**Lemma 2:**  $\mathcal{H}_{\hat{x}} = G'_{\hat{x}} - 1 < 0$ ;  $\mathcal{H}_r > 0$ ;  $\mathcal{H}_{\lambda} < 0$ ;  $\mathcal{H}_{\gamma} > 0$ ;  $\mathcal{H}_N + \mathcal{H}_{\hat{x}} < 0$ .

We then obtain:

**Proposition 3:** The amount of research performed in a positive balanced growth equilibrium will:

- (a) decrease with the rate of interest  $r$ ,
- (b) increase with the arrival parameter  $\lambda$ ,
- (c) increase with the size of each innovation (i.e. decrease with  $\gamma$ ), and
- (d) increase with the total labor endowment  $N$ .

This proposition is intuitive:

(a) an increase in the interest rate  $r$  means an increase in the required rate of return on research, whose effect will be to reduce the total investment in R & D.

(b) an increase in the arrival rate  $\lambda$  will have both the positive effect of raising the speed at which research pays (the effect on the numerator of 4.1), and the negative effect of increasing creative destruction (effect on the denominator). The first effect dominates.<sup>21</sup>

(c) an increase in the size of each innovation (decrease in  $\gamma$ ) also increases research by increasing the size of next interval's monopoly rents relative to today's research costs.

(d) an increase in total labor endowment  $N$  increases research by increasing, for a given  $\hat{n}$ , the size of the market that can be "monopolized" by a successful innovator.

We now complete our comparative statics analysis by studying the effect of the degree of monopoly power in the intermediate industry. In the Cobb–Douglas case, we saw that a good (inverse) measure of monopoly power was the parameter  $\alpha$ . In the general case we can proceed in a similar way and define:

$$(4.2) \quad F(x) = (f(x))^\alpha,$$

where  $0 < \alpha < 1$ . The elasticity  $\eta$  again depends positively on  $\alpha$ .<sup>22</sup>

We can then show:

**Lemma 3:**  $\mathcal{R}_\alpha > 0$ , where  $\mathcal{R}(\hat{x}, \alpha) \equiv G(\hat{x}, \alpha) - \hat{x}$ .

**Proof:**

$$\frac{\tilde{\pi}(\bar{x}^{-1}(\hat{x}))}{\bar{x}^{-1}(\hat{x})} = - \frac{1}{\eta(\hat{x}, \alpha) - 1}.$$

This expression decreases with  $\alpha$ . The lemma then follows from (4.1).  $\square$

We then obtain the following intuitive result:

**Proposition 4:** An increase in monopoly power (reduction in  $\alpha$ ) increases the amount of research in a balanced growth equilibrium.

We now derive an explicit expression for both the average growth rate (AGR) and the variability of the growth rate (VGR) in a balanced growth equilibrium. The volume of real output (i.e. the flow of consumption goods) during interval  $t$  is:

$$(4.3) \quad y_t = \frac{F(\hat{x})}{c_t},$$

which implies:

$$(4.4) \quad y_{t+1} = \gamma^{-1} \cdot y_t.$$

Thus the time path of the log of real output  $\ln y(\tau)$ —where  $\tau$  is real time—will be a random

step-function starting at  $\ln y_0 = \ln F(H, \hat{x}) - \ln c_0$ , with the size of each step equal to the constant  $-\ln \gamma > 0$ , and with the time between each step  $\{\Delta_1, \Delta_2, \dots\}$  a sequence of iid. variables exponentially distributed with parameter  $\lambda \hat{n}$ . This statement, together with (4.1), fully specifies the stochastic process driving output, as a function of the parameters of the model.

Not surprisingly, this stochastic process is nonstationary. Suppose observations were made at discrete points in time 1 unit apart. Then from (4.4):

$$(4.5) \quad \ln y(\tau+1) = \ln y(\tau) + \varepsilon(\tau); \tau = 0, 1, \dots$$

where  $\varepsilon(\tau)$  is  $-\ln \gamma$  times the number of innovations between  $\tau$  and  $\tau+1$ . It follows from the above discussion that  $\left\{ \frac{\varepsilon(0)}{-\ln \gamma}, \frac{\varepsilon(1)}{-\ln \gamma}, \dots \right\}$  is a sequence of iid. variables distributed Poisson with parameter  $\lambda \hat{n}$ . Thus (4.5) can be rewritten as:

$$(4.6) \quad \ln y(\tau+1) = \ln y(\tau) - \lambda \hat{n} \ln \gamma + e(\tau); \tau = 0, 1, \dots$$

with

$$(4.7) \quad e(\tau) \text{ iid.}, E(e(\tau)) = 0, \text{ var } e(\tau) = \lambda \hat{n} (\ln \gamma)^2$$

where  $e(\tau) \equiv \varepsilon(\tau) + \lambda \hat{n} \ln \gamma$ . From (4.6) and (4.7), the discrete sequence of observations on log output follow a random walk with constant positive drift. It also follows that:

$$(4.8) \quad \text{AGR} = -\lambda \hat{n} \ln \gamma, \text{ VGR} = \lambda \hat{n} (\ln \gamma)^2.$$

Combining (4.8) with Propositions 3 and 4 we can now sign the impact of parameter changes on AGR and VGR. Increases in the arrival parameter, the size of innovations, the size of labor endowment and the degree of monopoly power all raise AGR. Increases in the rate of interest lower it. These parameter changes have the same qualitative effect on VGR as on AGR. The effects are intuitive and straightforward. The effect of monopoly power, combined with our earlier finding (in the Cobb–Douglas case) that a minimal degree of monopoly power is needed before growth is even possible, underline the importance of imperfect competition for the growth process.



The effects also highlight another Schumpeterian theme: the tradeoff between current output  $y_t = F(\hat{x})/c_t$  and growth. Any parameter change (other than  $N$ ) that raises AGR also reduces  $y_t$  (taking  $c_t$  as predetermined), by drawing labor out of manufacturing. (In the case of  $\alpha$  that effect is amplified by the attendant shift of the production function in the consumption–good sector.) However, the loss of current output is not, as Schumpeter argued, a static efficiency loss of monopoly. Our assumption of inelastic labor supply prevents imperfect competition from putting the economy inside its current frontier of manufacturing and research. Whether the economy picks a good point on that frontier will be examined in the next subsection.

The positive effect of  $N$  on AGR has the unfortunate implication, which Romer (1988) has noted in a similar context, that larger economies should grow faster. In fact, (4.1') in footnote 20 above implies that, in the Cobb–Douglas case, a doubling of population should more than double the growth rate. (These comparisons must be intertemporal rather than international, since we have not dealt with the question of transboundary technology flows.) We accept the obvious implication that this class of models has little to say, without considerable modification, about the relationship between population size and growth rate.

The variability of output in this model should be distinguished from its variability in existing real business cycle models, for the following reasons. First, the variability of shocks has been endogenized here. An innovation in the statistical sense in real output is the effect of an innovation in the economic sense, the distribution of which depends as we have seen upon economic decisions that respond to parameter changes.

Second, output never falls in this model, contrary to what is observed in recessions, and contrary to what happens in existing real business cycle models. This is because all innovations are increments in knowledge. A negative  $e(\tau)$  indicates not a decrease in knowledge but a smaller than average increase. We accept the implication that the random fluctuations of output around the balanced growth path of our model tell a seriously

incomplete story of the business cycle. Instead they are best thought of as portraying the stochastic trend of output. (Output can fall, however, in a 2-cycle.)

### b. Welfare

We now compare the laissez-faire AGR derived above with the AGR that would be chosen by a social planner whose objective was to maximize the expected present value of utility of consumption  $y(\tau)$ . The maximized value of the planner's objective function,  $Z_t$ , when  $c_t$  is historically given, is determined by the following Bellman equation:

$$(4.9) \quad rZ_t = \max_{\{x_t \leq N\}} \left\{ \frac{F(x_t)}{c_t} + \lambda(N - x_t) \cdot (Z_{t+1} - Z_t) \right\}$$

The first-order condition an interior solution for this problem is:

$$(4.10) \quad F'(x_t)/c_t = \lambda(Z_{t+1} - Z_t)$$

It is easily checked that the solution to (4.9) is:

$$(4.11) \quad Z_t^* = \frac{Z_{t-1}^*}{\gamma} = \frac{F(x^*)/c_t}{r + \lambda(N - x^*)(1 - \gamma^{-1})},$$

where  $x^*$  solves:

$$(4.12) \quad \gamma = \frac{\lambda(1-\gamma) \cdot \left[ \frac{F(x^*)}{F'(x^*)} - x^* \right]}{r + \lambda N (1 - \gamma)^{-1}}$$

Thus the social optimum is a balanced growth path, with real output growing according to the same stochastic process as before, but with innovations arriving at the rate  $\lambda(N - x^*) = \lambda n^*$  instead of  $\lambda(N - \hat{x}) = \lambda \hat{n}$ . Accordingly laissez-faire produces an AGR more (less) than optimal if  $\hat{x} < (>) x^*$ .

Which way these inequalities go can be checked by comparing (4.1) and (4.12). The former can be rewritten as:

$$(4.13) \quad \gamma = \frac{\lambda \cdot \frac{\tilde{\pi}(\tilde{x}^{-1}(\hat{x}))}{\tilde{x}^{-1}(\hat{x})}}{r + \lambda(N - \hat{x})}$$

which, from the definition of  $\tilde{\pi}$  and  $\tilde{x}$ , can be rewritten as:

$$(4.14) \quad \gamma = \frac{\lambda(1-\gamma)(\frac{p}{w} - 1) \hat{x} + \lambda \gamma \frac{p}{w} \hat{x}}{r + \lambda N}$$

where  $\frac{p}{w} = \frac{F'(\hat{x})}{wc}$  is the constant ratio between price and marginal cost in the monopolistic intermediate industry on the balanced growth path. The right side of (4.14) is the private reward to research, deflated by next period's real wage. The right side of (4.12) is the corresponding social reward, deflated by the shadow cost of research next period.

There are three differences between (4.12) and (4.14): The first difference is the presence of the term  $-\lambda N \gamma^{-1}$  in the denominator of (4.12). This corresponds to the intertemporal spillover effect discussed in section 2(c). A private innovator will capture rent from his innovation for one interval only, whereas the social rent continues forever. Formally it comes from the presence of the term  $\lambda Z_{t+1}$  in the definition (4.9) of the social planner's value function  $Z_t$ , whereas no such term appears in the definition (2.17) of the private value of an innovation,  $V_t$ . This effect leads the laissez-faire economy to underinvest in research: i.e.  $\hat{x} > x^*$ .<sup>23</sup>

The second difference is the presence of the term  $\lambda \gamma \frac{p}{w} \hat{x}$  in the numerator of (4.14). This corresponds to a "business-stealing" effect. The private innovator does not internalize the loss of surplus to the monopolist whose rents he destroys. It comes formally from the presence of  $-\lambda(N - x_t)Z_t$  in the social planner's maximand in (4.9), whereas no corresponding subtraction appears in the problem (2.15) solved by the private research firm. This effect leads private firms to overinvest in research: i.e.  $\hat{x} < x^*$ .

The third difference is the presence of the term  $(\frac{p}{w} - 1)\hat{x}$  instead of  $[\frac{F(x^*)}{F'(x^*)} - x^*]$  in the numerator of (4.14). This corresponds to the monopolistic distortion effect. It arises because the flow of returns whose capitalized value the social planner attempts to maximize is total output, whereas the private value  $V_t$  is the capitalized value of profits,  $\pi_t$ . This distortion can work in either direction. In the Cobb-Douglas case it is non-existent, since in that case

$F/F' - x = (\frac{1-\alpha}{\alpha})x = (\frac{p}{w} - 1)x$ . (In the Cobb–Douglas case output and profits are proportional to one another:  $\pi_t = (1-\alpha)y_t$ , so the effect amounts to nothing more than a multiplicative constant in the social–planner's decision–problem.)

The intertemporal spillover effect will dominate when the size of innovations is large (i.e.  $\gamma$  is small)—in this case the private reward to research will be small compared to the big social reward—and laissez–faire will generate an AGR less than optimal.<sup>24</sup> On the other hand when there is much monopoly power ( $\alpha$  close to zero in the Cobb–Douglas case) and innovations are not too large (i.e.  $\gamma$  is not too small), the business–stealing effect will dominate, leading to an AGR under laissez–faire which exceeds the optimal level of average growth.<sup>25</sup> (This case cannot arise in Romer's (1988) model where there is no destruction of existing monopolistic activities).<sup>26</sup>

### c. Introducing Learning by Doing

We now introduce a second source of growth besides innovations: namely, the accumulation of learning–by–doing (lbd) in the intermediate industry. A natural way to formalize lbd is to assume that in the time interval between two successive innovations, the intermediate industry can still improve upon the quality of its intermediate goods through manufacturing them: more formally, if  $c_t(\tau)$  is the value of the unit cost parameter at time  $\tau$  in interval  $t$ , and if  $x$  is the amount of labor devoted to manufacturing intermediate goods at that instant, we assume:

$$(4.15) \quad \frac{\dot{c}}{c} = -g x$$

where  $g > 0$ . Thus, learning–by–doing in the intermediate industry will increase productivity in the consumption good sector at the rate  $g x$  between two innovations. When a new innovation occurs, productivity will jump as before by the factor  $\gamma^{-1}$ .

Following A. Young (1928) and more recently Arrow (1962), Romer (1986), and Dasgupta–Stiglitz (1988), we assume that the returns from learning by doing are shared by all

firms in the economy. In particular the intermediate firms experience a complete spillover of their lbd, which also spills over into the research sector. Because of this externality, each intermediate firm will choose its optimal level of intermediate output  $x(i)$  as before, relying for lbd on the other intermediate producers; that is, taking as given the economy-wide average  $x$  in (4.15). In Romer (1986), this type of externality leads private firms to underinvest in knowledge, hence to an average growth rate under laissez-faire which is less than the social optimum. In this paper, however, the spillover of lbd can generate the opposite result; i.e. that private economies grow too fast.<sup>27</sup>

Let  $V_t(\tau)$  denote the value to a research firm of making the  $t^{\text{th}}$  innovation at time  $\tau$ . Define  $x_t(\tau)$ ,  $w_t(\tau)$  and  $\pi_t(\tau)$  analogously. We confine attention to balanced growth equilibria, in which  $x_t(\tau) = \hat{x}$  (constant). Then  $\hat{x} = \tilde{x}(c_t(\tau)w_t(\tau))$  as before, so  $c_t(\tau)w_t(\tau) = cw$  (constant). Likewise  $\pi_t(\tau) = \tilde{\pi}(cw)/c_t(\tau)$ . As before,  $V_t(\tau)$  is the expected present value of  $\pi$  evaluated over interval  $t$ . Since  $\pi_t(\tau)$  grows at the constant exponential rate  $g\hat{x}$  over this interval, it follows that:

$$(4.16) \quad V_t(\tau) = \frac{\pi_t(\tau)}{r + \lambda\hat{n} - g\hat{x}}$$

Therefore:

$$(4.17) \quad V_t(\tau) = \frac{\tilde{\pi}(\tilde{x}^{-1}(\hat{x}))/c_t(\tau)}{r + \lambda N - (\lambda + g)\hat{x}}.$$

The first-order condition for positive research to occur is, as before,  $w_t(\tau) = \lambda V_{t+1}(\tau)$ . Thus the level of output  $\hat{x}$  in a (non-degenerate) SE is given by:

$$(4.18) \quad \gamma = \frac{\tilde{\pi}(\tilde{x}^{-1}(\hat{x}))/\tilde{x}^{-1}(\hat{x})}{r + \lambda N - (\lambda + g)\hat{x}}$$

This equation is identical to (4.1) except for the term  $-g\hat{x}$  in the denominator. Thus the previous analysis of stationary states was a special case, with  $g = 0$ .

The AGR under laissez-faire is given by:

$$\text{AGR} = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \cdot E(\ln y(\tau) - \ln y_0) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} (g\hat{x}\tau - E(t)\ln\gamma)$$

(where  $t$  is the number of innovations that occurred until date  $\tau$ .) Given our assumption that innovations follow a Poisson process with arrival rate  $\lambda \hat{n}$ , we have:

$$\lim_{\tau \rightarrow \infty} E\left(\frac{t}{\tau}\right) = \lambda \hat{n}.$$

Hence:

$$(4.19) \quad \text{AGR} = g(N - \hat{n}) - \lambda \hat{n} \ln \gamma.$$

In words, the average growth rate now derives from two components: a deterministic component due to lbd ( $g(N - \hat{n})$ ) and a random component corresponding to the innovation process ( $-\lambda \hat{n} \ln \gamma$ ). On the other hand, VGR is given by the same formula as before since the additional growth generated through lbd is deterministic:

$$(4.20) \quad \text{VGR} = \lambda \hat{n} (\ln \gamma)^2$$

By contrast with Section 4(a) above, parameters may affect AGR and VGR in opposite directions: for example, an increase in  $r$  will reduce the amount of research  $\hat{n}$  performed in equilibrium, thereby reducing VGR. The effect on AGR is ambiguous: the random part will

be reduced as before but the deterministic part will increase together with the amount of manufacturing labor  $\hat{x} = N - \hat{n}$ . In particular AGR will increase if  $g > -\lambda\eta\gamma$ .

An increase in the speed of learning by doing (i.e. in  $g$ ) will have a positive direct effect on AGR (equal to  $\Delta g(N - n)$ ). It will also indirectly affect AGR by increasing the level of research  $\hat{n}$ .<sup>28</sup> When the initial value of  $g$  is sufficiently small ( $-\lambda\eta\gamma - g > 0$ ), the indirect effect will increase AGR. When the initial  $g$  is large however, a further increase in  $g$  may have a perverse effect on growth by inducing too much research at the expense of learning by doing.

The tendency for lbd to induce too much research is brought out more clearly by the following welfare analysis. Let  $Z_t(\tau) = Z(c_t(\tau))$  be the social planner's value function. It satisfies the following Bellman equation, analogous to (4.9) above:

$$(4.21) \quad rZ(c_t(\tau)) = \max_{\{x \leq N\}} \left\{ \frac{F(x)}{c_t(\tau)} + \lambda(N-x) [Z(\gamma^{-1}c_t(\tau)) - Z(c_t(\tau))] - Z'(c_t(\tau))gxc_t(\tau) \right\}$$

The additional term  $-Z'(c_t(\tau))gxc_t(\tau)$  is the social return from learning by doing at time  $\tau$ , according to (4.15).

It is straightforward to verify that if research is positive then the solution to (4.21) is:

$$(4.22) \quad Z(c_t(\tau)) = \frac{1}{c_t(\tau)} \left[ \frac{\gamma F'(x^*)}{\lambda(1-\gamma) - g\gamma} \right],$$

where  $x^*$  solves the equation:

$$(4.23) \quad \gamma = \frac{(\lambda(1-\gamma) - g\gamma) \left( \frac{F(x^*)}{F'(x^*)} - x^* \right)}{r + \lambda N(1 - \gamma^{-1})}$$

The comparison between the equilibrium level  $\hat{x}$  under laissez-faire and the corresponding level  $x^*$  at the social optimum (i.e. between (4.18) and (4.23)) involves the same three effects as before, i.e. the intertemporal spillover effect, the business stealing effect and the monopolistic distortion. But there is now an additional source of distortion due to the introduction of learning by doing as a second factor lbd; terms in  $(-g)$  appear in the

denominator of (4.18) and in the numerator of (4.23): in other words introducing lbd increases research under laissez-faire, but reduces it in the social optimum! This effect reinforces the "business-stealing effect" mentioned earlier.

The economics of this result can be summarized as follows: learning by doing raises the reward to research activities, by raising the net present value of rents from a successful innovation. But it does not raise the marginal profitability of hiring manufacturing workers, since the benefits of lbd are external to the firm (our spillover assumption). This explains why a laissez-faire economy will respond to an increase in  $g$  by investing more in research at the expense of manufacturing. With the social planner things will go the other way around: internalizing the effect of lbd in his decisions, the social planner will respond to an increase in  $g$  by putting more workers into manufacturing, thus fewer workers into research.

To see that learning by doing can make a laissez-faire economy grow too fast, consider the following example. Start with an economy without learning by doing, where  $\hat{x} = \hat{x}(g = 0)$  is almost equal to  $x^* = x^*(g = 0)$ .<sup>29</sup> Then:

$$\begin{aligned} \frac{d}{dg} (AGR^* - AGR) &= \frac{d}{dg} (g + \lambda \ln \gamma) \cdot (x^*(g) - \hat{x}(g)) \\ &= x^* - \hat{x} + (g + \lambda \ln \gamma) \cdot \left( \frac{dx^*}{dg} - \frac{d\hat{x}}{dg} \right) \end{aligned}$$

As we have seen,  $\frac{dx^*}{dg} - \frac{d\hat{x}}{dg} \Big|_{g=0} > 0$ . Therefore, if  $x^*(0)$  is sufficiently close to  $\hat{x}(0)$ , introducing learning by doing will generate too much growth at the margin:

$$AGR^*(dg) < AGR(dg).$$

## 5. CONSUMPTION – SMOOTHING: MAKING INNOVATIONS LARGER CAN DETER RESEARCH

In this section we relax the assumption of constant marginal utility. Instead, the instantaneous utility function has a constant elasticity of marginal utility equal to  $\sigma > 0$ . Thus



$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$ . The pure rate of time preferences is still the constant  $r > 0$ . Thus the elasticity of intertemporal substitution is  $1/\sigma$ . The preceding analysis dealt with the special case of  $\sigma = 0$ . With  $\sigma > 0$ , consumers will want to smooth consumption over time, which will affect the equilibrium.

We confine attention to situations of non-degenerate balanced growth. All households are identical, so all will have equal consumption in equilibrium, equal to  $y/N$ . Thus each innovation will reduce the marginal utility of consumption by the factor

$u'(y_{t+1}/N)/u'(y_t/N) = \gamma^\sigma$ . Each shareholder of a research firm will thus have a marginal rate of substitution equal to  $\gamma^\sigma$  between the consumption good just before an innovation and the consumption good just after. Accordingly, a research firm will discount the payoff to a successful innovation by the factor  $\gamma^\sigma$ , and its marginal condition for positive research will be

$$(5.1) \quad w_t = \lambda \gamma^\sigma V_{t+1}$$

where  $V_{t+1}$  is the market value in terms of the consumption good of the  $t + 1^{\text{st}}$  innovation.

That value continues to be determined by (2.17).<sup>30</sup> From (5.1) and (2.17):

$$(5.2) \quad w_t = \frac{\lambda \gamma^\sigma \pi_{t+1}}{r + \lambda n_{t+1}}$$

The local monopolist's choice problem will be the same as before. Thus in a balanced growth equilibrium (5.2) implies:

$$(5.3) \quad \gamma^{1-\sigma} = \frac{\lambda \tilde{\pi}(\tilde{x}^{-1}(\hat{x}))/\tilde{x}^{-1}(\hat{x})}{r + \lambda(N - \hat{x})}$$

Equation (5.3) determines  $\hat{x}$ , and hence also the level of research  $\hat{n} = N - \hat{x}$ . The only difference between (5.3) and the previous equation (4.13) is the presence of  $\sigma$  on the left side. The average growth rate and its variability continue to be determined by (4.8). Thus all the comparative statics effects on  $\hat{n}$ , AGR and VGR remain unchanged except for the effect of the size of innovations.

Specifically, an increase in the size of innovations (decrease in  $\gamma$ ) still increases research, and hence increases the average growth rate and its variability, if the elasticity of

intertemporal substitution exceeds unity ( $\sigma < 1$ ). In this case the decrease in  $\gamma$  decreases the left side of (5.3), which causes a decrease in  $\hat{x}$ . But if the elasticity of intertemporal substitution is less than unity ( $\sigma > 1$ ), then an increase in the size of innovations leads to a reduction in research. If the elasticity is sufficiently close to zero this could lead to such a large reduction in research as to reduce AGR and VGR.

The economic interpretation of this new possibility is straightforward. A decrease in  $\gamma$  raises the payoff to research as measured in units of consumption good. But it also reduces the marginal utility of consumption after the innovation relative to before. When  $\sigma > 1$  the latter effect is so strong as to reduce the utility rate of return to research. The effect is similar to the negatively sloped savings schedule that occurs when  $\sigma > 1$  in the standard two-period overlapping generations model with all endowments accruing to the young.

An increase in the intertemporal elasticity of substitution (decrease in  $\sigma$ ) increases the amount of research, and hence also increases the average growth rate and its variability, because it reduces the left side of (5.3). Intuitively, it raises the utility-rate of return to research by reducing the rate at which marginal utility falls after an innovation.

The welfare analysis of balanced-growth equilibria is almost the same as before. The maximized value of the social planner's objective function  $Z_t$  is determined by the Bellman equation:

$$(5.4) \quad rZ_t = \max_{\{x_t \leq N\}} \left\{ \frac{1}{1-\sigma} [F(x_t)/c_t]^{1-\sigma} + \lambda(N - x_t)(Z_{t+1} - Z_t) \right\}.$$

It is easily verified that if research is positive then the solution to (5.4) is

$$Z_t^* = Z_{t-1}^* / \gamma^{1-\sigma} = F(x^*)^{-\sigma} F'(x^*) / \lambda(\gamma^{\sigma-1} - 1) c_t^{1-\sigma},$$

where  $x^*$  solves:

$$(5.5) \quad \gamma^{1-\sigma} = \frac{\lambda(1-\gamma^{1-\sigma}) \left[ \frac{F(x^*)}{F'(x^*)} - x^* \right]}{r + \lambda N(1 - \gamma^{\sigma-1})}.$$

Comparison of (5.5) with (5.3) reveals the same three differences as before between  $\hat{x}$  and  $x^*$ . So, as before, laissez-faire may produce either too much or too little research.

## 6. ENDOGENOUS SIZE OF INNOVATIONS: INNOVATIONS ARE TOO SMALL UNDER LAISSEZ-FAIRE

This section generalizes the analysis of balanced growth developed in section 4 by allowing research firms to choose not only the frequency but also the size of innovations. We show that the equilibrium size of innovations will be independent of everything in the model except the technology of research firms. Thus all the comparative-statics effects previously analyzed remain unchanged. There is, however, a change in the welfare analysis. Specifically, under laissez-faire innovations will be too small. This new effect will reinforce the intertemporal-spillover effect in tending to make the economy grow too slowly.

Rather than assume a linear research technology at the individual firm level, it is helpful to assume an infinitely elastic supply of identical research firms with U-shaped cost curves. Assume that to experience innovations with a cost reduction factor  $\tilde{\gamma}$  at the rate  $\lambda\tilde{z}$  a firm must hire  $v(\tilde{z}, \tilde{\gamma})$  units of labor, with  $v_1 > 0$ ,  $v_2 < 0$ . We look for a stationary equilibrium with a constant aggregate level of research employment  $n$ , with  $\tilde{z}$  and  $\tilde{\gamma}$  equal to the constants  $z$  and  $\gamma$  within each research firm, and with  $c_t w_t = cw = \text{constant}$  for all  $t$ . Since  $n/v(z, \gamma)$  is the number of firms, the aggregate arrival rate of innovations is  $[n/v(z, \gamma)]\lambda z = (\lambda/k)n$ , where  $k = v(z, \gamma)/z$  is a constant.

In a stationary equilibrium, the intermediate demand for manufacturing labor, i.e.  $x = N - n$ , is given by:

$$(2.5) \quad c_t w_t = cw = F'(x) + x F''(x),$$

and the value of the  $t+1^{\text{st}}$  innovation is:

$$(6.1) \quad V_{t+1} = \frac{\tilde{\pi}(cw)/c_{t+1}}{r + (\lambda/k)n} = \frac{\tilde{\pi}(cw)/cw}{r + (\lambda/k)n} w_{t+1} = A w_{t+1}$$

If a firm choosing  $\tilde{\gamma}$  makes the  $(t+1)^{\text{st}}$  innovation, it will make the wage rate rise to:

$$(6.2) \quad w_{t+1} = \frac{cw}{c_{t+1}} = \left(\frac{cw}{c_t}\right)\tilde{\gamma}^{-1} = w_t\tilde{\gamma}^{-1}.$$

During interval  $t$ , the research firm takes as given that  $V_{t+1}$  will be determined by (6.1) and (6.2), and takes  $A$  and  $w_t$  as given. Thus its decision problem is:

$$(6.3) \quad \max_{\{\tilde{z}, \tilde{\gamma}\}} -w_t v(\tilde{z}, \tilde{\gamma}) + \lambda \tilde{z} A \tilde{\gamma}^{-1} w_t.$$

Assume that (6.3) has an interior solution. By free entry, the maximized expression in (6.3) must equal zero. From this and the first-order conditions:

$$(6.4) \quad v_1 = v\tilde{z}^{-1}$$

$$(6.5) \quad v_2 = -v\tilde{\gamma}^{-1}$$

From (6.3) the output of research is proportional to  $\tilde{z}/\tilde{\gamma}$ . Therefore a research firm's average cost is proportional to:

$$K(z, \gamma) \equiv \gamma v(z, \gamma)/z.$$

Our assumption of U-shaped cost is:

Assumption A.3:  $K(z, \gamma)$  is strictly convex.

It follows from (A.3) that (6.4) and (6.5) define a unique equilibrium combination  $(\hat{z}, \hat{\gamma})$ ; specifically, the combination that minimizes average cost:

$$(6.6) \quad (\hat{z}, \hat{\gamma}) = \arg \min K(z, \gamma).$$

Since none of the parameters  $(r, \lambda, N)$  enter (6.6), it follows that  $\gamma$  and  $z$  are independent of them. Similarly, the size of each innovation is independent of the degree of monopoly power. Thus all comparative-statics results go through as before except for those describing

the effects of varying the size of innovations, which are no longer relevant since the size is endogenous.

To conduct the welfare analysis, note that the social planner's maximized objective function  $Z(c_t)$  is given by the Bellman equation:

$$(6.7) \quad r Z(c_t) = \max_{\{z, \gamma, m\}} \left\{ \frac{F(N - mv(z, \gamma))}{c_t} + \lambda m z (Z(\gamma c_t) - Z(c_t)) \right\}$$

where  $m$  is the number of firms.<sup>31</sup>

The first-order conditions for this problem are:

$$-F' v c_t^{-1} + \lambda z (Z(\gamma c_t) - Z(c_t)) = 0$$

$$-F' c_t^{-1} m v_1 + \lambda m (Z(\gamma c_t) - Z(c_t)) = 0$$

$$-F' c_t^{-1} m v_2 + \lambda z m c_t Z'(\gamma c_t) = 0$$

It is easily checked that the solution to (6.7) is:

$$Z(c_t) = \gamma^{*-1} Z(c_{t-1}) = \frac{F(N - n^*)}{r - (\lambda/k^*)(\gamma^{-1} - 1)n^*} c_t$$

where  $k^* = v(z^*, \gamma^*)/z^*$ ,  $n^* = m^* v(z^*, \gamma^*)$  and:

$$(6.8) \quad v_1 = v z^{*-1}$$

$$(6.9) \quad v_2 = -(1 - \gamma^*)^{-1} v \gamma^{*-1}$$

Our main result is that innovations are too small in equilibrium:

**Proposition 5:**  $\hat{\gamma} > \gamma^*$ .

**Proof:** Define  $a \equiv \gamma^* v(z^*, \gamma^*)/z^* (1 - \gamma^*) > 0$ . From A.3, (6.8) and (6.9):

$$(6.10) \quad (z^*, \gamma^*) = \arg \min \{K(z, \gamma) + a\gamma\}.$$

From (6.6) and (6.10):

$$\begin{aligned} K(\hat{z}, \hat{\gamma}) &< K(z^*, \gamma^*), \\ K(\hat{z}, \hat{\gamma}) + \alpha \hat{\gamma} &> K(z^*, \gamma^*) + \alpha \gamma^*. \end{aligned}$$

From these inequalities:  $\alpha \hat{\gamma} > \alpha \gamma^*$ . Since  $\alpha > 0$ , therefore  $\hat{\gamma} > \gamma^*$ .  $\square$

This result is another manifestation of the business-stealing effect. The smaller the innovation the larger the losses through obsolescence relative to the gains of an innovation. The private firm ignores this cost of raising  $\gamma$ .

By free entry the maximand in (6.3) equals zero. This implies:

$$(6.11) \quad \hat{\gamma} = \frac{\tilde{\pi}(\tilde{x}^{-1}(\hat{x})) / \tilde{x}^{-1}(\hat{x})}{\hat{k}r/\lambda + N - \hat{x}},$$

where  $\hat{k} = v(\hat{z}, \hat{\gamma})/\hat{z}$ . The solution to (6.7) yields:

$$(6.12) \quad \gamma^* = \frac{(1-\gamma^*) \left[ \frac{F'(x^*)}{F'(x^*)} - x^* \right]}{k^* r/\lambda + (1-\gamma^{*-1}) N}.$$

Comparison of (6.11) with (6.12) reveal the same three distortions as before on the equilibrium amount of research. In addition, the fact that  $\gamma > \gamma^*$  will tend to create too little research. The fact that  $k^* \neq \hat{k}$  will also generate a distortion. As before, the total level of research may be too much or too little.

## 7. RANDOM ARRIVAL PARAMETER: CREATIVE DESTRUCTION CAN DESTROY GROWTH

In this section we allow the arrival parameter  $\lambda$  to vary randomly. This generalization illustrates the force of creative destruction by allowing an increase in  $\lambda$  in some states of the world to deter research in other states. In fact, we present an example where in the limit as the value of  $\lambda$  in one state becomes infinite, the average growth rate falls to zero.

Let  $\{\lambda_1, \dots, \lambda_m\}$  be the finite set of possible values of  $\lambda$ . At the moment of any innovation a new  $\lambda$  is drawn, according to the transition matrix  $A$ , and all research firms learn

this value. Transition into a high- $\lambda$  state could represent a fundamental breakthrough that leads to a Schumpeterian wave of innovations, whereas transition to a low state could represent the exhaustion of a line of research.

The stochastic equivalent of balanced growth equilibrium is an equilibrium in which manufacturing employment depends only on the state of the world, not on time. Let  $V_j/c_t$  be the value of making the  $t^{\text{th}}$  innovation and moving into state  $j$ . In any state  $i$ , the marginal expected return to research in interval  $t$  is  $\lambda_i \sum_{j=1}^m a_{ij} V_j/c_{t+1}$ . This will equal the wage if positive research occurs in state  $i$ . If research occurs in all states, the  $V_i$ 's must satisfy the Bellman equations:

$$(7.1) \quad rV_i = \tilde{\pi}(\lambda_i \sum_j a_{ij} V_j / \gamma) - \lambda_i [N - \tilde{x}(\lambda_i \sum_j a_{ij} V_j / \gamma)] V_i ; i=1, \dots, m$$

The AGR equals  $-f \ln \gamma$ , where  $f$  is the asymptotic frequency of innovations. Define  $n_i \equiv N - \tilde{x}(\lambda_i \sum_j a_{ij} V_j / \gamma)$ . Then:

$$(7.2) \quad f = \sum_{i=1}^m \lambda_i n_i q_i$$

where  $q_i$  is the asymptotic fraction of time spent in state  $i$ .

Our example has  $m = 2$  and  $a_{ij} = 1/2 \forall_{ij}$ . It is easily verified that in this case:

$$(7.3) \quad q_1 = 1 - q_2 = \frac{\lambda_2 n_2}{\lambda_1 n_1 + \lambda_2 n_2}$$

From (7.2) and (7.3):

$$(7.4) \quad f = \frac{2 \lambda_1 \lambda_2 n_1 n_2}{\lambda_1 n_1 + \lambda_2 n_2}$$

To complete the example, take the Cobb–Douglas case ( $F(x) = x^\alpha$ ) and suppose  $\alpha = \gamma = 1/2$ , and  $r = N = \lambda_1 = 1$ . Using (2.10) and (2.12), (7.1) can be rewritten as:

$$(7.5) \quad \begin{cases} V_1 = [16(V_1 + V_2)]^{-1} - \{1 - [4(V_1 + V_2)]^{-2}\} V_1 \\ V_2 = [16\lambda_2(V_1 + V_2)]^{-1} - \lambda_2 \{1 - [4\lambda_2(V_1 + V_2)]^{-2}\} V_2 \end{cases}$$

When  $\lambda_2 = 1$ , the solution to (7.5) is  $V_1 = V_2 = \sqrt{3}/8$ , which implies  $n_1 = n_2 = 1/3$ , and  $f = 1/3$ . When  $\lambda_2 = \infty$ , the solution is  $V_1 = 1/4$ ,  $V_2 = 3/4$ , which implies  $n_1 = 0$ ,  $n_2 = 1$ , and  $f = \frac{2n_1 n_2}{n_1/\lambda_2 + n_2} = 0$ . Thus raising the productivity of research in one state can cause research to be discouraged in the other state, through the threat of creative destruction, to such an extent that growth is eliminated.

## 8. CONCLUSION

We have presented a model of economic growth based on Schumpeter's process of creative destruction. Growth results exclusively from technological progress, which in turn results from competition among research firms that generate innovations. Each innovation consists of a new line of intermediate goods that can be used to produce final output more efficiently than before. Research firms are motivated by the prospect of monopoly rents that can be captured when a successful innovation is patented. But those rents in turn will be destroyed by the next innovation, which will render obsolete the existing line of intermediate goods.



The model possesses a unique balanced-growth equilibrium, in which there is a constant allocation of labor between research and manufacturing. In that equilibrium the log of GNP follows a random walk with drift. The size of the drift is the economy's average growth rate, and the variance of the increments is the variability of the growth rate. Both these parameters of growth are endogenous to the model, and depend upon the rate of time preference and elasticity of intertemporal substitution of the representative household, and upon the size and likelihood of innovations resulting from research.

As Schumpeter argued, the degree of market power available to someone implementing an innovation will also be an important determinant of growth. We have parametrized the degree of market power, and shown that it has a positive effect upon both the average growth rate and its variability.

The average growth rate and its variability are also affected by the extent of learning-by-doing in the manufacturing sector of the economy. The greater the coefficient of learning-by-doing, the more research will be undertaken. This will raise the average rate of growth attributable to research, but it may decrease the growth attributable to learning-by-doing, because the only way for society to engage in more research is to take labor out of manufacturing.

The average growth rate may be more or less than socially optimal, because there are two counteracting distortions. The first is a technology spillover. A successful innovation produces knowledge that other researchers can use without compensation to the innovator. The counteracting distortion is a "business-stealing" effect. The private research firm does not internalize the loss to others due to obsolescence resulting from his innovations. We also show that innovations are too small under laissez-faire, again because of the business-stealing effect.

Learning by doing introduces a further distortion because it is external to the individual manufacturing firm. This distortion tends to produce too little manufacturing, and hence too much research. Whether it produces too much or too little growth depends upon the relative

efficacy of learning by doing and research as sources of growth.

Under some circumstances the model possesses equilibria in addition to the unique one with balanced growth. One is a "no-growth trap" in which the economy stops growing in finite time because of the (rational) expectation that if one more innovation were to take place it would be followed by a flurry of research activity. This expectation makes it so likely that the rents accruing to the producer of the next innovation will be destroyed quickly that it is not worth trying to create the innovation, and research stops. Another possible equilibrium is a real 2-cycle, in which manufacturing employment oscillates between two values. These cycles, as well as those arising from random imitation or a random arrival parameter of innovations, have the interesting implication of a negative correlation between the cyclical components of productivity ( $y_t c_t / N$ ) and real wages ( $w_t c_t$ ), because  $y_t c_t / N = F(\tilde{x}(w_t c_t)) / N$ . Relatively high real wages arise from a relatively high incentive to research, and discourage the output of goods.

Two final results are worth mentioning here. First, if the representative household in the model has a sufficiently low intertemporal elasticity of substitution then an increase in the size of innovations can actually reduce the amount of research, and hence reduce the average rate of growth in the economy. Second, when the Poisson arrival rate of innovations is a random function of the amount of research, an increase in the likelihood of innovations in one state of the world will result in extra creative destruction in that state; this can discourage research in other states to such an extent that the economy's average growth rate falls.

We conclude by noting directions for future research. It would be useful to allow the size of innovation eventually to fall, to allow for the possibility that technology is ultimately bounded. Also to include a richer intersectoral structure to study both the positive dynamics of diffusion and the normative effects of intersectoral spillover. The model would also gain

richness and realism if capital were introduced, either physical or human capital embodying technical change, or Rand D capital that affects the arrival rate of innovations.<sup>32</sup> Allowing unemployment by introducing search externalities into the labor market, and changing the structure of demand for intermediate goods so as to allow for a contemporaneous aggregate demand externality, might also generate multiple equilibria and allow us to study the reciprocal interaction between technical change and the business cycle. All these extensions seem feasible because of the simplicity and transparency of the basic model.

## APPENDIX 1

Assumption A.1 is satisfied in the CES case.

Let  $F(x) = (x^\rho + H^\rho)^{1/\rho}$ , where  $0 < \rho < 1$ .

We have:

$$F' = 1/\rho (x^\rho + H^\rho)^{1/\rho-1} \rho x^{\rho-1} = x^{\rho-1} \cdot (x^\rho + H^\rho)^{1/\rho-1}$$

Hence:

$$\begin{aligned} F'' &= (\rho - 1)x^{\rho-2}(x^\rho + H^\rho)^{1/\rho-1} + x^{\rho-1} \cdot (1/\rho - 1)(x^\rho + H^\rho)^{1/\rho-2} \rho x^{\rho-1} \\ &= (\rho - 1)(x^\rho + H^\rho)^{1/\rho-2} [x^{\rho-2}x^\rho + x^{\rho-2}H^\rho - x^{2\rho-2}] \\ &= (\rho - 1)(x^\rho + H^\rho)^{1/\rho-2} x^{\rho-2} H^\rho < 0 \end{aligned}$$

and:

$$\begin{aligned} \frac{1}{H^\rho} \cdot F''' &= (\rho - 1)(1/\rho - 2)(x^\rho + H^\rho)^{1/\rho-3} \rho x^{\rho-1} x^{\rho-2} \\ &\quad + (\rho - 1)(x^\rho + H^\rho)^{1/\rho-2} (\rho - 2)x^{\rho-3} \\ &= (\rho - 1)(x^\rho + H^\rho)^{1/\rho-3} [(1 - 2\rho)x^{2\rho-3} + (\rho - 2)x^{2\rho-3} + H^\rho x^{\rho-3}(\rho - 2)] \\ &= (\rho - 1)x^{\rho-3}(x^\rho + H^\rho)^{1/\rho-3} [(\rho - 2)H^\rho - (1 + \rho)x^\rho]. \end{aligned}$$

Therefore:

$$\begin{aligned} 2F'' + xF''' &= (x^\rho + H^\rho)^{1/\rho-3} H^\rho (\rho - 1) [2x^{\rho-2}(x^\rho + H^\rho) \\ &\quad + x^{\rho-2}[(\rho - 2)H^\rho - (1 + \rho)x^\rho]] \\ &= (\rho - 1) \cdot H^\rho \cdot (x^\rho + H^\rho)^{1/\rho-3} [x^{\rho-2}][x^\rho(1 - \rho) + \rho H^\rho] \\ &< 0 \end{aligned}$$

□

## APPENDIX 2

Derivation of (3.7)

We have:

$$\mathcal{J}'(\hat{x}) = \frac{d}{dx} \left[ \tilde{x} \left[ \frac{\tilde{\pi}(\tilde{x}^{-1}(\hat{x}))/\gamma}{r/\lambda + N - \hat{x}} \right] \right].$$

Using (2.5), (2.8), and the fact that  $\hat{x} = \tilde{x}(wc)$ , we get:

$$\begin{aligned} \mathcal{J}'(\hat{x}) &= \frac{\tilde{x}'[wc]}{r/\lambda + N - \tilde{x}(wc)} \left[ \frac{\tilde{\pi}'(wc)/\gamma}{\tilde{x}'(wc)} + \frac{\tilde{\pi}(\tilde{x}^{-1}(x))/\gamma}{r/\lambda + N - x} \right] \\ &= \frac{1/\gamma}{r/\lambda + N - \tilde{x}(wc)} (\tilde{\pi}'(wc) + \gamma wc \tilde{x}'(wc)) \\ &= \frac{wc}{\tilde{\pi}(wc)} [-\tilde{x}(wc) + \gamma wc \tilde{x}'(wc)] \\ &= \frac{x(wc) \cdot wc}{\tilde{\pi}(wc)} (-\gamma \eta_{wc}^x - 1) \\ &= (\eta - 1)(-\gamma \eta_{wc}^x - 1), \end{aligned}$$

where  $\eta_{wc}^x = -\frac{wc \tilde{x}'(wc)}{\tilde{x}(wc)}$ .

By the first-order condition (2.5):

$$F'(x)(1 - \frac{1}{\eta(x)}) = wc.$$

Differentiating this w.r.t. (wc), we can reexpress  $\eta_{wc}^x$  as:

$$\eta_{wc}^x = \frac{\eta - 1}{1 - 1/\eta - \frac{x\eta'}{\eta}}$$

Hence:

$$\mathcal{J}'(\hat{x}) = (\eta - 1) \left[ \frac{\gamma(1 - \eta)}{1 - 1/\eta - \frac{x\eta'}{\eta}} - 1 \right].$$

□

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## FOOTNOTES

<sup>1</sup>In the Deneckere–Judd model a constant proportion of existing intermediate goods disappear each period. The equilibrium of the model exhibits no growth.

<sup>2</sup>More recent work by Cochrane (1988) shows that this evidence is not, however, conclusive.

<sup>3</sup>Thus policy analysis based on the King–Rebelo model would be subject to the Lucas critique because it treats the variability of growth as invariant to the policy determinants of growth.

<sup>4</sup>Mention should also be made of the preliminary work of Corriveau (1988), who is developing a similar model of growth and cycles in a discrete time setting. The discreteness of time in Corriveau's model introduces complications from the possibility of simultaneous innovations, which we avoid by our assumption of continuous time. In Corriveau's framework all innovations are non-drastic, and it is not possible to parameterize the degree of monopoly power in the economy as we have done.

<sup>5</sup>Some consequences of introducing memory are discussed in section 8 below.

<sup>6</sup>Scherer (1984) combines process- and input-oriented R and D into a measure of "used" R and D, which he distinguishes from pure product R and D. He estimates that during the period 1973–1978 in U.S. industry the social rate of return to "used" R and D lay between 71% and 104%, whereas the return to pure product R and D was insignificant.

<sup>7</sup>Gradual diffusion could be introduced by allowing the cost parameter after each innovation to follow a predetermined but gradual path asymptotically approaching the limit  $c_t$ , and then to jump to  $c_t$  upon the next innovation and follow a gradual path approaching  $c_{t+1}$ . This would produce a cycle in research within each interval, as the gradual fall in costs induces manufacturing firms to hire more and more workers out of research until the next innovation occurs.

<sup>8</sup>Note that this description of technology implies increasing returns in the production of the consumption good, the ultimate inputs of which are land, labor (as embodied in the intermediate goods) and knowledge (as embodied in the  $c$ 's). If land and labor alone were doubled, then the flow of intermediate input would also double, and hence, by the assumption of constant returns, final output would just double. Since there are constant returns in these two factors holding the third constant, there are increasing returns in all three.

The technological specification of the economy follows that of Griliches (1979). There exist empirical studies casting doubt on our assumptions that there are constant returns in research in that a doubling of research will double the rate of innovations (Griliches, 1989), that there is no variability in the size of innovations (Pakes, 1986), and that there is no variability in the Poisson arrival rate of innovations (Hausman, Hall, and Griliches, 1984). The first two assumptions can be relaxed with only notational difficulties. The third is relaxed in section 7 below.

Finally, we could modify the intermediate technology (2.2) to allow a specialized fixed factor in addition to labor. We could interpret the specialized factor as unskilled labor, and, interpret the labor that can be used in either manufacturing or research as skilled labor, thus adding plausibility to the model. The work of Romer (1988) suggests that no substantive difficulties would be created.

<sup>9</sup>The alternative assumption that research firms have identical U-shaped functions with a small efficient scale produces identical results. This alternative is employed in section 6 below.

<sup>10</sup>Bellman equations like (2.14) are standard in the patent–race literature (see Tirole,

1988, ch. 10). Several more are introduced below. This footnote presents an alternative derivation. Let  $T$ ,  $T'$ , and  $T''$  be, respectively, the time of the next innovation, the time at which the other firms would experience their first innovation if they held employment constant at  $\bar{n}_t$  forever, and the time at which this firm would experience its first innovation if it held its employment constant at  $z$  forever. Under the no-spillover assumption,  $T'$  and  $T''$  are independent random variables, exponentially distributed with respective parameters  $\lambda\bar{n}_t$  and  $\lambda z$ , and  $T = \min(T', T'')$ . It follows that, as already mentioned,  $T$  is exponentially distributed with parameter  $\lambda(\bar{n}_t + z)$  and that the probability that the next innovation will be made by this firm, conditional on the date of that innovation,  $\text{Prob}\{T = T' | T\}$  is the constant  $z/(\bar{n}_t + z)$ . Thus, conditional on  $T$ , the value of the research firm is:

$$W_t(T) = e^{-rT} \left[ W_{t+1} + \text{Prob}\{T=T' | T\} V_{t+1} \right] - \int_0^T e^{-rt} w_t z d\tau$$

Thus:

$$W_t(T) = e^{-rT} \left[ W_{t+1} + (z/(\bar{n}_t + z)) V_{t+1} \right] - \frac{1}{r} (1 - e^{-rT}) w_t z$$

By definition:

$$W_t = \text{Max}_{\{z \geq 0\}} E W_t(T).$$

Since  $E(e^{-rT}) = \lambda(\bar{n}_t + z)/(r + \lambda(\bar{n}_t + z))$ :

$$W_t = \text{Max}_{\{z \geq 0\}} \frac{\lambda(\bar{n}_t + z) W_{t+1} + \lambda z V_{t+1} - w_t z}{r + \lambda(\bar{n}_t + z)}.$$

The last equation is equivalent to (2.14).

<sup>11</sup>It is straightforward to relax the assumption of complete spillover by allowing the  $t^{\text{th}}$  innovator to have an advantage in finding the  $t+1^{\text{st}}$  innovation; to have an arrival parameter  $\lambda' > \lambda$ . We have worked out this analysis under the assumption of U-shaped cost with small efficient scale (see footnote 9), with no important change in results.

<sup>12</sup>Our analysis of research is derived from the patent-race literature surveyed by Reinganum (1987) and Tirole (1988, ch. 10). Within that literature the paper closest to ours is Reinganum (1985), which emphasizes the affinity to creative destruction. Most other papers do not have the sequence of races necessary for creative destruction. Our paper goes beyond that literature by embedding the sequence of races in a general-equilibrium setting. Thus (a) the flow of profits  $\pi_{t+1}$  resulting from an innovation will depend upon the amount of research during the next interval, (b) our welfare criterion is the expected lifetime utility of the representative consumer, (c) we characterize the time-series properties of GNP, and (d) in section 5 we find that the size of innovations affects the rate of discount applied to research. In addition, the analysis in section 6 of the endogenous determination of the size of

innovations, and the analysis of cycles in section 3, go beyond anything we have found in the patent-race literature.

<sup>13</sup>G is well defined on (0,N), by A.2.

<sup>14</sup>The methods of Azariadis (1981) or Woodford (1986) could also be used to construct rational-expectations sunspot equilibria, at least in some cases.

<sup>15</sup>This distinguishes our model from the growth model with externalities developed in Romer (1986), which may have no steady state.

<sup>16</sup>When  $r=0$ ,  $x^c$  is defined by:

$$\bar{x} \left[ \frac{\tilde{\pi}(\bar{x}^{-1}(x^c))/\gamma}{N - x^c} \right] = N;$$

whereas

$$G(N) = \bar{x} \left[ \frac{\tilde{\pi}(\bar{x}^{-1}(N))/\gamma}{0} \right] = \bar{x}(+\infty) = 0 < x^c, \text{ by A.2.}$$

By continuity, (3.6) will hold for  $r$  (or  $r/\lambda$ ) positive but small.

<sup>17</sup>A real 2-cycle also exists if  $G(N) > x^c$  and  $|\mathcal{J}(\hat{x})| > 1$ . However, this can never occur in the Cobb-Douglas case (see Aghion and Howitt, 1988).

$$\eta(x) \equiv - \frac{F'(x)}{xF''(x)}.$$

<sup>19</sup>Likewise, the reason for cycles is similar to that underlying the chaotic fluctuations in the model of Deneckere and Judd (1986). They model innovations as Romer (1988) does, with no creative destruction. Their dynamics are backward looking, because the incentive to innovate depends upon the existing number of products, which depends upon past innovative activity, whereas their assumption that monopoly rents are not destroyed by future innovations makes the incentive independent of future innovative activity. Their cycles arise because the only rest-point of their backward-looking dynamics is unstable.

<sup>20</sup>In the Cobb-Douglas case, (4.1) can be expressed by:

$$(4.1'): \quad \gamma = \frac{\lambda \cdot \frac{1-\alpha}{\alpha} \cdot \hat{x}}{r + \lambda(N - \hat{x})} = \frac{\frac{1-\alpha}{\alpha} \cdot (N - \hat{n})}{r/\lambda + \hat{n}}$$

<sup>21</sup>In the extension of the model considered in section 7 the second effect may dominate.

$$\eta(x, \alpha) \equiv -F'(x)/xF''(x) = \frac{f(x) f'(x)}{(1-\alpha)x[f'(x)]^2 - xf(x)f''(x)}.$$

<sup>23</sup>Two additional kinds of spillover can easily be included. First, researchers could benefit from the flow of others' research, so that an individual firm's arrival rate would be a constant returns function  $\lambda(z, n)$  of its own and others' research. Second, there could be an exogenous Poisson arrival rate  $\mu$  of imitations that costlessly circumvent the patent laws and clone the existing line of intermediate goods. Both would have the effect of lowering AGR. Also as we showed in Aghion and Howitt (1988), the inclusion of  $\mu$  would introduce another source of cycles in the economy, since each imitation would make the intermediate industry perfectly competitive, which would raise manufacturing employment, until the next innovation

arrives.

<sup>24</sup>More formally, as  $\gamma$  falls to the lower limit  $\frac{\lambda N}{r + \lambda N}$ ,  $x^*$  approaches zero, whereas  $\hat{x}$  approaches a strictly positive limit.

<sup>25</sup>In the Cobb–Douglas case, if  $\frac{1-\alpha}{\alpha} > r/\lambda N$ , then  $\hat{n} > 0$ , for all  $\gamma$  (see 3.5) whereas  $n^* = 0$  when  $\frac{\gamma}{1-\gamma} > \lambda N/\alpha r$ .

<sup>26</sup>There seems to be a presumption among empirical students of R and D that spillovers will dominate business stealing in a welfare comparison. See, for example, Griliches (1979, p. 99).

<sup>27</sup>The formal source of these different results is that in our model research is a separate activity from the production of the goods embodying knowledge. The activity whose doing creates external learning is the production. Doing it more intensively requires less research to be undertaken. Romer follows Arrow (1962) in assuming that the two activities are inseparably bundled, thereby eliminating this tradeoff.

<sup>28</sup>This follows immediately from equation (4.18), using the fact that both  $\tilde{\pi}$  and  $\tilde{x}$  are decreasing function.

<sup>29</sup>In the Cobb–Douglas case:  $\frac{(1-\alpha)N - \alpha\gamma r/\lambda}{(1-\alpha) + \alpha\gamma} \simeq \frac{(1-\gamma)N - \alpha\gamma r/\lambda}{(1-\gamma)(1-\alpha)}$ .

<sup>30</sup>Someone who gave up a marginal unit of consumption to buy shares in a local patent would receive a constant dividend of  $\pi_t/V_t$  until  $T$ , the date of the next innovation. The expected utility gain of the transaction would be:

$$E_T \int_0^T u'(c_t)(\pi_t/V_t) d\tau = \frac{u'(c_t)\pi_t/V_t}{r + \lambda n_t}.$$

Equating this to the expected utility cost  $u'(c_t)$  yields (2.17).

<sup>31</sup>We assume that no non–negativity constraints bind.

<sup>32</sup>We have examined the consequences of a limited kind of R and D capital by allowing for the arrival parameter  $\lambda$  to increase randomly in the middle of an interval, and then to remain high until the next innovation, so that the probability of an innovation depends not just upon the flow of research but also upon the length of time over which the innovation has been sought. The analysis is formally like that of section 7 above, and most of the comparative–statics results of section 4 go through.